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ABSTRACT

The construction of internal representations in the domain of mathematics education is conceived as a signification process in this paper. Contrary to the established representation theory, it does not distinguish between an externally represented world and an internally representing world. Representation is regarded as a process in which new "signs" are constantly emerging by means of continuous and cyclic signification. Consequently, an internal representation ("signifier") transforms and is the basis ("signified") for the construction of a new internal representation ("signifier"). Hence a person constructs internal, mental representations on the basis of internal representations. This concept has some implications for the instruction model in that teaching mathematics is not to be seen merely as a process of transmitting knowledge. Children construct basic, internal representations demanding interactive testing. This external dialogue leads to reflection or internal dialogue. On the basis of reflection, representations on a higher level are developed and, successively, these new constructions demand new dialogue again. Higher levels of representation are not attained on the basis of interaction alone, but on the basis of what interaction evokes, i.e. reflection. It is for this reason that socio-constructivistic theory should pay more attention to reflection, because the process of level elevation can be better understood in this fashion. Finally, the paper discusses the relationship between constructivism/socio-constructivism and the notion of realistic mathematics education. Along with the differences, one essential similarity between theorists is emphasized: in both theories, mathematization is conceived of as a process of progressive signification. Meaning or 'common sense' is the beginning and the end of learning mathematics. Contains 74 references. (Author/DDR)

Representations in Mathematics Education

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Running Head: REPRESENTATIONS IN MATH EDUCATION

Abstract

The construction of internal representations in the domain of mathematics education is conceived as a signification process in this paper. Contrary to the established representation theory, it does not distinguish between an externally represented world and an internally representing world. Representation is regarded as a process in which new 'signs' are constantly emerging by means of continuous and cyclic signification. Consequently, an internal representation ('signifier') transforms and is the basis ('signified') for the construction of a new internal representation ('signifier'). Hence a person constructs internal, mental representations on the basis of internal representations.

This concept has some implications for the instruction model in that teaching mathematics is not to be seen merely as a process of transmitting knowledge. Children construct basic, internal representations demanding interactive testing. This external dialogue leads to reflection or internal dialogue. On the basis of reflection, representations on a higher level are developed and, successively, these new constructions demand new dialogue again. Higher levels of representation are not attained on the basis of interaction alone, but on the basis of what interaction evokes, i.e. reflection. It is for this reason that socio-constructivistic theory should pay more attention to reflection, because the process of level elevation can be better understood in this fashion.

Finally, the paper discusses the relationship between

constructivism/socio-constructivism and the notion of realistic mathematics education. Along with the differences, we would also like to emphasize one essential similarity between theorists: in both theories, mathematization is conceived of as a process of progressive signification. Meaning or 'common sense' is the beginning and the end of learning mathematics (Freudenthal, 1991).

KEYWORDS: realistic math education, socio-constructivism, cognitive representation

Representations in Mathematics Education

The idea that people form representations in their mind is one with a long tradition. The history of science shows how time and again, new images representing the current state of scientific insight have been constructed (Coplestone, 1985; Dijksterhuis, 1975). People want to clarify to each other what they understand their world to be. Consequently, representations come into being by a process of co-construction and are constantly being tested and criticized by the community (Sinha, 1988).

The concept of representation is at the forefront of cognitive psychology (DeLoache, 1989). Although the concept was rejected by some scholars in the past as psychologically useless (Gibson, 1966), others considered representation to be meaningful action (Bruner, 1974, 1996; Sinha, 1988). In their critical analyses, constructivistically oriented mathematicians or mathematically oriented constructivists (Von Glasersfeld, 1991) interpret the relationship between representation and what is represented in such a way that the internal experiences are represented instead of 'reality'. Internal representations are therefore the issue, and this paper is referring to this concept when the term representation is used.

Representations are not considered to be a direct reflection of the world. Human knowledge is constructed knowledge (Phillips, 1995); mathematical insights and procedures are not discovered, but invented, i.e. devised by people (Freudenthal, 1983). The position that representations are constructions has implications for the instruction model. This infers that children are to be looked upon as active thinkers, an idea which changes the image of the learning child. According to Bruner (1996), the child should be regarded both as learner and as epistemologist. The teacher's task is to look for the roots of systematic knowledge in the child's intuition.

The aim of the present article is to investigate what is understood by the concept of representation in mathematics education. The 'classical' concept of representation will be compared with the view upheld by 'situated-cognition' theoreticians. What does a constructivistic view of representation mean for the didactics of mathematics education and what are the consequences of that view for the instruction model? The instruction model advocated in this article will be compared with the socio-constructivistic view of mathematics education. It will be argued that there should be a greater focus on the concept of reflection in the above-mentioned theory, so that the transition to higher representation levels can be explained more adequately. On this basis, the paper will examine what in mathematics education are considered to be meaningful representations. Finally, the relationship between constructivism and realistic didactics of mathematics education will be analyzed.

Mathematical Representations as Means of Structuring

In cognitive psychology, theoreticians - following the logician Frege (1848-1925) - distinguish between 'referent' and 'sense.' The concept 'referent' refers to what is being represented - external reality - while the concept 'sense' refers to how something is represented. 'Sense' is representation as such (Palmer, 1978; Kosslyn, 1978; Dretske, 1986; Bechtel & Abrahamsen, 1991). Where iconic representations ('images' Kosslyn, 1994) as well as propositional representations (Anderson, 1990) are concerned, representation refers to a represented, external world. However, the discussion regarding the nature of the relationship between 'referent' and 'sense' is still ongoing (Sinha, 1988).

The way in which mathematics didacticians view the concept of representation is closely linked to their vision of mathematics. Mathematics is thus not the 'absolutistic' science it has long been considered to be. It is no longer viewed as a ready-made construction characterized by indisputable knowledge, but as an invention of man. That is what the history of mathematics shows; mathematicians were constantly engaged in a debate and theorems were found to be neither useful nor justifiable in the long run. Mathematics originates in human activity; it is not discovered, but intentionally invented. People devise mathematical means to order all sorts of phenomena - 'phainomena' says Freudenthal (1983) - with which they are confronted in culture (think of counting, measuring and localizing; Bishop, 1988). We can regard those ordering means to be 'noöemena' (Freudenthal, 1983), mathematical representations. So L x W is a formal representation of the phenomenon surface area. The formula sur = L x W can be seen as an organization of measurement of an area of right-angled figures operating as a representation of area. Freudenthal interprets 'noöemena' as mathematical structures. The didactics of realistic mathematics education is founded on these basic assumptions (Treffers, 1987).

Formation of Internal Representations

To represent: a process of signification

Prior to describing the process leading to the formation of internal representations, we will first examine further the difference between external and internal representations. For as far as the didactics of mathematics education is concerned, external representations are ready-made visualizations, schemata, models and tables, in short, concrete representations designed by the teacher (or developer) using the mathematical properties of representation for the purpose of teaching children to work with them. Examples of such representations are the number line, the abacus (with its position system) and the proportion table. In addition, models constructed by mathematicians are considered external representations (for example algebraic or graphic models), without which scientific debate would be inconceivable and which perform an indispensable function in society as 'cultural tools.'

Internal representations are constructed on the basis of meaningful experiences, also called mental representations (Fodor, 1981). External representations are created from meaningful experiences, as internal representations are; consequently, there is no difference in this respect. The difference is particularly didactic in nature, in the sense that external representations have been constructed by adults on the assumption that children will, from their point of view, understand the mathematical notions constructed from the adult angle. Internal representations, however, are in principle self constructed by children, which increases the chance that the action is meaningful and insightful for the children.

According to the linguist Halliday (1978), the child initially understands its world on the basis of meaning, which it constructs. At first, cognitive development is neither directed by thinking and logic as described by Piaget (1977) nor by language (Vygotsky, 1977) according to Halliday. Thinking and language are directed by the construction of meaning. Halliday (1978) characterizes his approach as 'social semiotic.' By taking this particular approach, he intends both to analyze the development of meaning and to understand it against the background of the social environment in which the child grows up and in which meaning comes into existence (see among others Sinha, 1988). According to Walkerdine (1982), context 'in the mind' comes into existence on the basis of meaning. The child forms 'models' (Nelson 1996), schemata or scripts of often-experienced situations which have come to be meaningful for the child. Action occurs in contexts, in situations; it is situated, and therefore in this respect the term 'situated cognition' is applied (Kirshner & Whitson, 1997). This means that not only do people think in different contexts, but they also act differently in various 'practices.' Different 'relations of signification' arise each time, according to Walkerdine (1997). She advocates what she calls a psycho-semiotic analysis of 'signs,' i.e. a method of analysis for investigating social processes in which 'signs' arise.

Walkerdine takes the thought that network relationships between significants are created from Lacan, who speaks of a 'semiotic chain.' As this notion and the concept 'sign' are essential for our line of reasoning, we will pursue this issue in greater detail. A 'sign' consists of two constituents, a 'signifier' and a 'signified' (Whitson, 1997) which form a unity, as represented schematically in Figure 1.

INSERT FIGURE 1 ABOUT HERE

New 'signifiers' are generated repeatedly in a process characterized by Whitson (1997) as 'chaining of signifiers' and illustrated as follows:

INSERT FIGURE 2 ABOUT HERE

Initially we see five persons, each of whom has a name. The names are the 'signifier', the persons are the 'signified.' Subsequently, the five names can be represented by five fingers. 'Signifier' now becomes 'signified' and the child devises a new 'signifier,' i.e. five fingers. At an even higher level, the five fingers become 'signified' and the new 'signifier' consists further of spoken number symbols: one up to and including five (number symbols are used in the next phase of the process). What happens, is that a new 'sign' arises again and again, i.e. a new combination of 'signifier' and 'signified', while each time a 'signifier' is constructed at a higher level. However, the new 'signifier' does not refer to an object in the external world, but to a 'signifier' constructed earlier. The child does not represent an objective reality but a mental, internal process; he/she re-presents his/her own internal representations recursively (Nunes, 1992). Subsequently, these internal representations affect the perception of reality, or to use Sinha's (1988) words, representations are constitutive for the material world.

The process by which an internal representation is formed, is nearly always connected with a social context and experiences which take on meaning in that particular context. In other words, representation processes are 'situated' and consequently everything has or takes on meaning (words, tools, signs, and so on) according to Lemke (1997). He speaks of 'ecosocial systems,' and argues against isolating other social systems, practices and contexts from each other. After all, human action always stands for operating with 'signs' (characterized as 'semiosis' by Whitson, 1997). The concept 'sign' is at the heart of this approach.

This 'situated cognition' view of the concept of representation differs fundamentally from the 'classical' theory of representation (see, for instance, Palmer, 1978; Dretske, 1986; Finke, 1989; Perner, 1991). The latter theory assumes a separation between a 'referent,' the objective external world represented, and a 'sense,' the internal representation in someone's mind. Essentially, this is a dualistic, and, according to Whitson (1997), a structuralistic view. This dualism has been superseded in 'situated cognition' theory, in that the 'signifier' always results from a preceding 'signifier,' i.e. is closely connected with meaningful internal experiences and, consequently, is not directly dependent on an existing external world. Or, according to Sinha (1988: 33): 'the "signified" is produced, rather than referred to.' One might say that the new 'signifier' organizes the actual experiences. The question is not whether there is an external, objective world; the discussion is more epistemological than ontological. The question is how do we <u>know</u> the external world.

In connection with 'situated cognition' theory, this article argues for a non-dualistic view of representation. A characteristic feature of this view is that in a constantly progressing and iterative signification process, a 'sign' is transformed into a 'signified' for a new 'signifier' at a higher level. Both in the realistic didactics of mathematics education (Freudenthal, 1991, Gravemeijer, 1995) and in socio-constructivistic theory (Cobb, Gravemeijer, Yackel, McClain & Whitenack, 1997), mathematization is essentially considered to be a signification process of this kind. According to Freudenthal (1991: 9), mathematics should start with the student's 'common sense.' In the words of Freudenthal: "Common sense, in order to become genuine mathematics and in order to progress, had to be systematized and organized. Common sense experiences, as it were, coalesced into rules (such as the commutativity of addition), and these rules again became common sense, say of a higher order" (Freudenthal, 1991: 9). In other words, mathematics starts and ends with childrens' meaning.

In a recent experiment, Cobb et al. (1997) demonstrated how the process of mathematization evolves as a signification process. The theoretical basis for the interpretation of the experiment was Walkerdine's (1988, 1997) signification theory, socio-constructivism (Cobb, 1994) and the realistic didactics of mathematics education (Gravemeijer, 1995). According to these researchers, the basic principles of their theories are largely consistent with each other: emphasis on student activity, creativity, problem solving, the reality of contexts and particularly 'mathematical reality,' meaning the creation of mathematical objects, thus of a new reality. They therefore feel that an attempt at integration is both sensible and advisable. The authors describe at great length how children repeatedly construct new 'signifiers' - departing from the context of a 'candyshop' - and, among other things, use an arithmetic rack (with a five structure) and unifix blocks (consisting of ten bars and loose blocks). 'Candies' were represented by blocks, the blocks-representing-'candies' by pictures, etc. In other words, 'chains of signification' were created as shown in Figure 3 (derived from Cobb et al., 1997: 192).

INSERT FIGURE 3 ABOUT HERE

Levels of Representation

'Common sense' is organized into mathematics on a more formal level. This is shown in the process by which children acquire their first counting experiences, on the basis of which they start structuring and proceed to formal counting. Freudenthal (1991) calls the ability to recite the counting sequence the first algorithm of a mathematical nature mastered by the child through 'common language,' the language with which it is confronted in meaningful situations (Nelson, 1996). The development of the process in which the child gets a grip on whole numbers can be chronologically outlined as follows. The child is able to recognize quantities of two and three, it learns the counting sequence as if it were a recited rhyme, it is able to identify a small quantity, to recognize number - symbols and count resultatively. The ensuing transition, i.e. from counting to structuring, is crucial because the insightful mental arithmetic proceeding from this is the foundation for all mental arithmetic operations the child as yet has to learn. As the child discovers meaningful structures, such as number images on a dice or the five fingers, counting one by one is reduced. Using such structures, quantities can be ordered and compared with each other. Adding up, first executed by counting, is now carried out on the basis of a new 'signifier': structure and number image. The child has constructed a basis for working according to rules: structure becomes 'signified'. The flexibility of addition as well as subtraction procedures increases as meaningful strategies are applied, for example working with 'double images' (3+3, 5+5). A condition for the whole process of emerging numeracy is that the child is offered stimulating experiences in meaningful situations (Nelissen, 1998).

This example shows that earlier 'common sense' insights undergo extensive restructuring through the repeated construction of new meaningful internal representations: from counting to structure and from structure to insightful application of rules. The addition of numbers, first considered and represented as a procedure performed by counting one by one, transforms into an insightful process using handy rules.

Fractions, too, are represented and understood at different levels (see Bokhove, 1996, Streefland, 1991). Initially, children work with informal, context-bound representations; for example, children start to measure a table using the unit of measure 'a foot' and they find the table to be 3 1/2 feet wide. Next is the semiformal, schematic, model-supported representation of fractions; for instance, the children show in a schema in how many ways three baguettes can be shared between four children. At the highest level of representation, formal, mathematical connections with related concepts become visible; children solve problems like "compare 3/4 with 0.7." The children now operate at a purely numeric-symbolic level of representation, and at this level their insight into the phenomenon of fractions changes radically.

However, the transition to higher levels of representation is often a source of problems in mathematics education (Janvier, 1987). One important cause of these problems is that mathematical denotations are confused with their everyday use (Zepp, 1989). According to Pimm (1991), this not only concerns matters like spoken language ambiguity, but also the ways in which children see something or "students' perspectives." In particular, when it comes to symbolic representations, confusion often arises and regression to a natural language and "everyday" thinking threatens.

The well-known 'Students Professor Problem' (Clement, 1982) illustrates such confusion. It is a very sticky problem, obviously not intended for elementary school students. The problem is the following: a university has one professor for six students. The assignment is to enter the ratio of students to professors in an algebraic equation. Over half (65%) of the participants (including adults) to whom the problem was put responded: 6S = P, the 'reversal error' according to Kaput (1987). The gap between the algebraic equation and everyday language was probably difficult to bridge: 'Six times more students' usually makes 6S. By applying a mathematical representation model, i.e. a table, we can get a grip on the problem.

INSERT TABLE 1 ABOUT HERE

Now, one is less inclined to compare persons instead of numbers. Table 1 is concerned with ratios and shows that 6 x P equals S, in short: 6 P = S.

Another example. Upon asking children which number is exactly between 0.9 and 0.11, the answer given is often 0.10. It is a matter of interference - more specifically, negative transfer between two systems of representation, i.e. the system of natural numbers (also used in everyday language) and the system of decimal numbers. For that reason, many authors favor paying attention to the transition from one mode of representation into another (Janvier, 1987; Kaput, 1987). Still, in which way does such a transition have to be elaborated to take the didactics of mathematics education into account?

The transition from one level of representation to another can be fostered by systematically appealing to reflection and also by stimulating students' own constructions and meaningfulness. The intention is to keep the signification process going as described above. The following two sections advocate an instruction model which highlights the importance of reflection.

Consequences for the Instruction Model

In this paper representations are considered to be meaningful constructions and this way of thinking has implications for the instruction model. First, teaching mathematics is not merely a matter of transmission of knowledge and second, children must be given the opportunity to elaborate their own constructions. It might be better to speak of co-constructions (Leseman & Sijsling, 1996), for the point is essentially - as explained in this section - how children arrive at mathematical activities together. The course of a teaching-learning process will be explained by means of an example.

In a maths class, children were asked to find out which of several differently shaped bottles (without labels) had the largest volume. In this experiment (see Nelissen & Tomic, 1994), the children made all sorts of discoveries: plunging bottles in the water, weighing, measuring by means of a cup, emptying bottles and seeing which produced the biggest puddle. There was much discussion about the different constructions or ideas and the children were encouraged to defend their own ideas. Moreover, they had to listen well to the arguments of others (Elbers, 1993, Mercer, 1995 and others showed the importance of these conversation characteristics). Asking the children which suggestion appealed to them most and why provoked curiosity and interaction, i.e. critical verification. In the next phase, verification turned into an experiment and the most appealing suggestions were tested. The experiences gained during experimentation were evaluated and the children were asked to critically reanalyze their own strategy on the basis of the discussion with and criticisms of the other children. The construction and interaction phase was followed now by a reflection phase. As intended, the childrens' own constructions were raised to a higher level by reflection. Of course, not all children reached a higher level, as some stuck to their initial procedure. However, in other children reflection really brought about representation at a higher level. If reflection indeed brings about new constructions at a higher level, the whole cycle can recommence: the new construction again causes critical interactive verification. Because of its cyclic nature, the teaching-learning process, as outlined above, can be characterized as dialectic.

In short, the instruction model can be described in general terms as follows. In the first phase, children form (co-) constructions on the basis of their prior knowledge. These provoke discussion and critical verification - often stimulated by teacher interventions. Through confrontation with other ideas and arguments during discussion, the need arises for critical reflection and, if necessary, adjustment of one's own representations on the basis of these ideas and arguments. Since reflection develops from dialogue, reflection can be characterized as internalized dialogue; external dialogue becomes internal dialogue. Also, anticipating communication with another person should be understood as an internalized dialogue (Nelissen & Tomic, 1996). Reflecting on its own actions will enable the child to construct representations at a higher level which in turn will demand critical retesting (Nelissen, Ruyters & Van Heest, in press).

Level Raising Through Reflection

In socio-constructivistic theory (Cobb & Bauersfeld, 1995; Cobb, 1994; Cobb et al., 1997; Gravemeijer, 1995) mathematization is regarded to be a process of signification in which internal representations develop at an increasingly higher level. How do these authors explain this rise in level? To pursue this question further, we will first look at the results of research on reflective thinking in elementary school children (Nelissen, 1993).

This research shows that children solving problems at an advanced level did not hesitate to change their strategies, were not afraid to abandon a once-chosen solution mode, looked for links with their general knowledge of the world, controlled spontaneously their quest for a solution, compared two problem solving procedures, and, in advance, steered themselves mentally towards possible procedures. These are important features of reflective thinking.

Poor problem solvers often held on to a standard procedure. For example, a picture shows a star-spangled sky. The children were asked approximately how many stars were shown in the picture. Poor arithmeticians opted for and stuck to a counting strategy, which they did not exchange for a smarter estimation strategy, even after pointed hints from the researcher. These children did not reflect at all, or only at a low level. For example, they were unable to compare two different procedures, not even their own procedure with one of their classmates.

Nevertheless, the comparison of two procedures is of great importance. In their educational experiments with children, Cobb et al. (1997) deliberately provoked comparison operations. For instance, when a student found an elegant solution offering good prospects, his classmates were not meant to blindly adopt his discovery. In the discussion which the teacher encourages among the children, we may assume that a child conceptualizes an image of the nature, intention and possible advantages of the new suggestion and compares that image with his or her own approach. This reflection process is essential for employing new procedures at a higher level, as emphasized in Piaget's (1977) and Freudenthal's (1979) work.

In Piaget's (1977) view, reflection ('réfléchissements') is a prerequisite for raising a child's level. 'Reflexive abstractions' are concepts on a constantly higher level developed on the basis of 'réflexions,' which Piaget considers to be 'moteurs du développement cognitif' (1977: 307). According to Freudenthal (1979: 9), reflection leads to mathematization at a higher level: 'The mathematics performed, acted out on a lower level, is made conscious and analyzed and consequently transformed into subject matter at a higher level'. Looking for the right term for this process, says Freudenthal, 'I hit on "reflection".'

From External to Internal Dialogue:

Socio-Cognitive Conflict

In Vygotsky's (1977) theory, reflection - like every higher mental function - develops from a dialogue between a child and adults. For this reason reflection may be called 'internalized dialogue'. The development of reflection seems inextricably connected with communication. One cannot help but notice that in socio-constructivistic views, reference is actually being made to Vygotsky (Cobb, 1994) and the analysis of the classroom communication structure is highlighted, while the development of internal dialogue (reflection) out of external dialogue is largely ignored. Yet the Vygotskian reflection view is well suited to socio-constructivism, because in both theories communication is seen as a prerequisite for the development of higher cognitive processes.

Just like Vygotsky, Freudenthal (1979) sees reflection as developing from interaction: 'mirroring oneself in the other in order to look through his skin.' 'There is one argument why reflective behavior should start with mirroring at the other's mind. The argument is language, or more generally, communication' (1979: 10).

However, the transition to higher representation levels does not stem directly from interaction and exchange of ideas, but from what the interaction evokes, i.e. reflection. The development of higher representation levels is mediated by reflection; if not, there is a risk that someone else's idea is being passively adopted. An important reflective moment is when one compares one's own approach with someone else's (possibly better) approach. It is very possible that such reflective processes took place in the minds of the children participating in the educational experiments reported by Cobb et al. (1997). However, these researchers analyzed the discussion between students from a sociological point of view, and their analyses clearly show the social norms shaping the discussion and the group-dynamic structure characterizing the social interaction.

As indicated by the authors, a psychological analysis of mental processes at the individual level is absent (1997: 214). Cobb is not always interested in such processes. In another recently conducted experiment (Cobb, 1997), the pupils did not have to learn to handle consciously 'smart' approaches ('consciously figure out', says Cobb, our italics), but were supposed to gain a clear insight into the relationships between numbers. Records show that high-quality interactions did occur, but that no conflict was elicited, a major reason for reflecting on one's own procedure and a major incentive for critically reconsidering one's own procedure if required. Students did occasionally accept a smart method from a fellow student, but from the records we cannot easily infer precisely on what grounds the acceptation is based. John says (p. 168): 'I get it...That's a good way.' How, then, according to John, did the new approach differ from his own approach? The researchers are most certainly in the process of detecting reflection, but it is not identified as such. 'Jordan appeared to reconceptualize his prior counting solution ...making the transition from counting by ones to grouping...' (p. 172). Quite likely, Jordan gives evidence of reflection. After all, he does reconsider and revise his own method and thus he reaches a higher level of mathematization.

The fact that individual students establish 'chains of signification' can, in our view, be explained by the individual reflection processes (see also Fogarty, Perkins & Barell, 1992).

According to the socio-constructivistic view, mathematization can be described schematically as follows: construction ('signifier' 1) --> interaction --> construction ('signifier' 2), etc. We suggest extending this schema as follows: construction ('signifier' 1) --> interaction --> reflection --> construction ('signifier' 2), etc.

The question remains: how can reflection be evoked during the process of mathematization? As various researchers have emphasized (Doise & Mugny, 1984; Borkowski, 1985; Kilpatrick, 1985), the socio-cognitive conflict can be used as a didactic strategy for evoking reflection. According to Doise and Mugny (1984), a socio-cognitive conflict occurs while communicating during social interaction. A person is faced with a dilemma by another person, discovers that other perspectives and solutions can be at issue and through these the need arises for the comparison of different perspectives. The original procedure or method is considered from another point of view, or someone else's point of view, according to Freudenthal (1979) 'shifting one's standpoint.' Socio-cognitive conflict may be evoked in different ways, for instance: (1) by challenging childrens' responses; (2) by introducing problems including several possible solutions; (3) by solving open-ended problems; (4) by utilizing Socratic phrasing of a question; (5) by solving unsolvable problems due to missing data.

In addition to reflection, the transition to higher representation levels is also fostered by taking into account a student's own sense-giving, as will be described in the next section. Besides reflection, there is yet another mechanism to achieve higher levels of representation.

'Common Sense', Concrete and Concrete-Material Representations

As a starting point for the process of mathemization, consideration is often given to the students' concrete start. However, the concept 'concrete' can be interpreted in two ways: firstly, as 'material-concrete' and, secondly, as experience. According to Freudenthal (1991), the starting point of mathematization is experience and he terms this 'common sense'. That is why one can speak of 'common sense representations' (Gravemeijer & Nelissen, 1996). The initial basic representations conceived by children are founded on experiences that are meaningful for children. The act of counting illustrated how emerging numeracy progresses, repeatedly achieves a higher level, and at each level turns into 'common sense.' In the realistic didactics of mathematics education (Treffers, 1987; Goffree, 1986), this is called vertical mathematization in context; at that level children represent from context-derived situation models and on the basis of these models vertical concept acquisition develops as well as representations such as tables, schemas, etc. According to Gravemeijer (1994), at the first level of meaning the child forms 'models of' which are comparable to the 'mental models' described by Johnson-Laird (1983). These models refer to context problems and informal strategies and convert into more general models, 'models for' which are representations of mathematical strategies for general mathematical reasoning.

In both cognitive psychology and mechanistic and structuralistic didactics of mathematics education, one relies on the realization of mathematical knowledge in external representations. As distinguished from realistic situation models, these representations are called 'material-concrete', whereas realistic models are called 'common-sense concrete'. 'Material-concrete' representations are considered to be embodiments of mathematics in its purest form. An example: the concept surface area is represented by a rectangle in which the number of columns must be multiplied by the number of rows. Obviously, this procedure is mathematically correct, though it is actually derived from the formal final stage of a standard calculation for surface area (L x W).

In Galperin's learning theory (1980), the learning process is founded on materialization (material action). Material action takes place when there is external intervention in concrete reality (the manipulation of objects); mental action takes place when this intervention is internal (Tomic & Kingma, 1996). The essence of Galperin's method of stepwise formation of mental actions is the formation of valuable mental actions based on material actions. What is materialized for the children, can be considered concrete-material representations. 'Common sense' is not the child's intention, but it is supposed that the structure of what is materialized guides the childrens' actions. In other words, basic representations are material-concrete and not 'common sense' in nature.

The concept of deducing representations directly from mathematics is characteristic for the works of Dienes (1963) and Piaget (1977), partly for Davydov's (1977) experimental programs and for the didactical elaboration of the 'information processing approach' (Resnick & Ford, 1981). The latter researchers in particular based their teaching on actions derived from experts.

Broadly speaking, material-concrete representations are often artificial, such as, for instance, the Cuisenaire-material, which consists of little bars of different colors and lengths, used as models for associating numbers. Few children succeed, particularly because they have no knowledge of the underlying structure and, thus, do not know what is actually represented (Cobb, Yaeckel & Wood, 1992). After all, the underlying structure was invented and made concrete by adults, who already have an insight into the system concretized by them. Consequently, they understand the embodiments. For children, the above-mentioned procedure makes no sense (Janvier, 1987) for they do not know what those embodiments refer to. This approach, therefore, provoked some criticism (Cobb et al., 1992; Greeno, 1991; Von Glasersfeld, 1991). Learning mathematics, the researchers argued, is essentially children forming internal representations of their experiences on their own. Facing them with ready-made external representations inhibits that process.

Cobb et al. (1992) characterize this didactic as the 'instructional representation approach' giving rise to the 'learning paradox' (Bereiter, 1985). While embodiment is meant to clarify an 'idea' or 'theory,' knowledge of that particular 'idea' or 'theory' is required for understanding what has been made concrete. To prevent a 'learning paradox,' teaching has to be based on meaningful contexts for children, enabling them to give meaning to their actions and to develop their own representations. This does not mean that representations introduced by teachers are wrong by definition, but that even in those cases children must be given the opportunity to develop representations alone or together while being guided by the teacher's hints. To a great extent, teachers are responsible for evoking the interactions and reflections that are essential for the construction process.

Constructivism and Realism

In this paper the concept of representation was analyzed from a socio-constructivistc point of view. However, what is the relationship between constructivism and the most prominent pedagogical movement, realistic educational theory (Goffree, 1986; Treffers, 1987)? According to Phillips (1995), constructivism is supported by a wide philosophical or theoretical spectrum, including Von Glasersfeld, Kant, Kuhn, Piaget and Dewey.

Originally, constructivism was a general theory of knowledge acquisition instead of an educational theory (Gravemeijer, 1995). However, realistic educational theory is a teaching methodology and, consequently, we are working with two different theories. While constructivism is characterized by a diversity of interpretations (Gravemeijer, 1995), the critical commentaries on constructivism are mainly aimed at radical constructivism (see for instance Solomon, 1994).

This paper focused on the rather young research tradition endeavoring to integrate socio-constructivism with realistic educational theory (Cobb, 1994, 1997; Gravemeijer, 1995). According to Gravemeijer (1995), these theories are complementary and research findings (Cobb et al., 1997) appear to support this view. However, the above-mentioned theories are different, not only because they were developed in a different theoretical context (epistemological versus teaching methodological) but also because they are aimed at differently programmed goals. Among the major goals of realistic mathematics educational research are: (1) research in the didactic macro-structure of mathematics education for the development of new curricula, (2) research into new learning routes fostering concept formation processes in the long term, (3) research into relationships between learning routes, (4) the development of student learning materials and teacher aids, (5) reflection on a 'national curriculum' for mathematics education, etc.

The socio-constructivist research program is not primarily directed by the demands from actual practice, but by the need to explore new perspectives in relatively small-scale research settings with a view to stimulating the development of theories. Perhaps these explorations cannot immediately be used to maximum advantage in educational practice, but they might well be fruitful in the long run. Socio-constructivists wish to demonstrate that children often learn more insightfully to mathematize on the basis of their own internal representations than on the basis of explanations and materializations developed for them by adults. This does not imply that socio-constructivists take the view that children have to invent everything by themselves; this ideal is mainly advocated by radical-constructivists. In those instances in which adherents to the realistic didactics of mathematics education react reservedly to constructivism, they most probably react against the ideal of the radicals. For the time being, De Lange (1997) speaks of 'minor differences,' mostly relating to the fact that the importance of the goals of realistic mathematics educational research as mentioned above are underestimated (see also Solomon, 1994), although they are a particularly important condition for successful educational reform.

In spite of the obvious differences between constructivism and realistic mathematics education mentioned by De Lange (1977) - and the ones he does not mention such as the extent of teacher control - there are also major similarities such as the attention to discussion and language, the importance of informal procedures, the function of contexts and the attention for motivation and meaning. However, of overriding importance is that socio-constructivism considers mathematization to be a process of advancing signification and this view on learning mathematics in particular is a tenet in Freudenthal's didactic phenomenology, i.e. mathematics in children always starts and ends with 'common sense.' Mathematics starts and ends for children with meaning. There is no disagreement concerning this issue, according to Dolk (1997).

Finally, the question remains what the importance of socio-constructivism is for everyday educational practice. Solomon (1994) warns against exaggerated expectations. Children are no scientists, and they do not construct knowledge in the way scientists do. What children have to learn is often totally new for them, and therefore their environment intuition and the new concepts that must be learned are unrelated. It is for this reason that Solomon warns against underestimating the teacher's role in the process of culture transfer (Van Oers, 1994). With this critical observation, Solomon (1994) turns against the radical constructivists.

Up to now Cobb et al. (1997) have shown in small-scale experiments the importance of socio-constructivism for the practice of mathematics education. However, the children were not asked to construct their own mathematics. The experiments investigated how pupils, through interaction with their fellow-pupils and intensively trained teachers, constructed mathematically relevant procedures (Cobb et al., 1992).

Conclusion and a Look Ahead

Socio-constructivism wrestles with two crucial problems. Firstly, it is as yet not certain that childrens' findings offer long-term possibilities for what is to be learned and secondly, one wonders how the teacher is to evaluate this process. Teacher interference is inevitable if a child's findings are not robust enough and without possibilities, and this inhibits the power of the constructivistic source of inspiration. To cope with this problem, it is necessary for the teacher to gain an insight into the macro-curricular position and organization of mathematics material, the 'big ideas' (Cobb, 1997), or beacons for the teacher to set his/her course by. Socio-constructivists have indeed recognized the need for a 'pedagogical agenda' for gaining insight into the 'potential significance of issues that might emerge as topics of conversations' (Cobb, 1997: 277).

The same holds for the instruction model presented in this paper. The model aims at evoking interaction and reflection on the basis of teacher intervention. This is possible on the basis of the macro-structure of every realistic curriculum at least in principle. Nevertheless, many, if not all, crucial activities will depend on the quality of teacher-student interactions (see Mercer, 1995). These must be high quality, i.e. the teacher must provoke and stimulate the desired processes in children at the right moment, in the right way, with the right means and with a clear insight into which findings of children do indeed offer prospects for mathematization. Accomplishing this will require intensive post-graduate teacher training.

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TABLE 1

Students Professor Problem (Clement, 1982)

S (number of students)	P (number of professors)
6	1
12	2
18	3

Figure Captions

Figure 1. Sign Constituents

Figure 2. Example of a Signification Chain

Figure 3. "Chain of Signification"

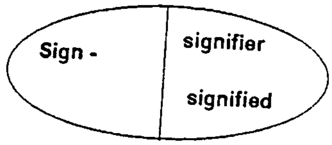

Figure 1

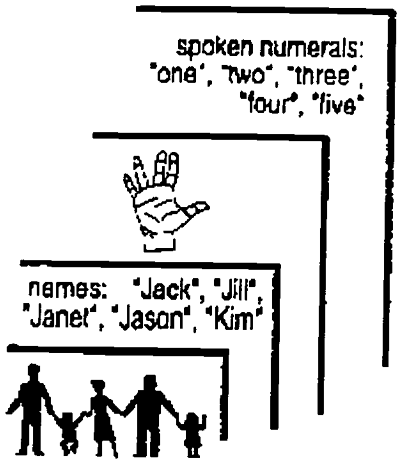
spoken numerals:
"one", "two", "three",
"four", "five"
names: "Jack", "Jill",
"Janet", "Jason", "Kim"

Figure 2

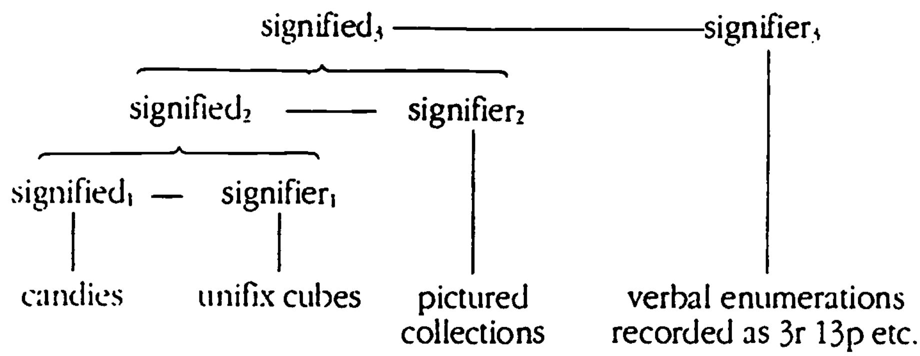
signified3
signifier3
signified2
signifier2
signified1
signifier1
candies
unifix cubes
pictured collections
verbal enumerations recorded as 3r 13p etc.

Figure 3

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